**Мarkov renewal theorem in the series scheme**

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**Introduction.**

The classical renewal theory deals with the asymptotic properties of the solutions to the renewal equation

where is the function to be found, is given function, and is a given probability distribution. The classical renewal theorems describe the asymptotic properties of convolutions

where is the potential of a homogeneous critical kernel , which is an ordinary probability distribution.

 The basic statements of the classical renewal theory can be extended to the so-called Markov renewal equation

where is a given phase space, is so-called semi-Markov kernel, is a given function of , and , and is the function to be found. Its solution is the convolution

where is the potential of the semi-homogeneous kernel .

Generally, the renewal theory has wide range of applications in mathematical practice. Markov renewal theorems are an analytical tool for studying the limiting behavior of Markov and related processes, including semi-Markov and regenerative processes.

**Main results.**

Let (*E*,) be a measurable (phase) space with the countably generated σ-algebra . We will assume, without loss of generality, that σ-algebra contains all one-point sets. Let us introduce a family of non-negative semi-homogeneous [3] kernels which depend on a small parameter .

Consider the Markov renewal equation

where is a given nonnegative +-measurable function, is the function to be found, + is the Borel σ-algebra on *R+*.

Next, we impose a number of restrictions. Let's assume that the kernels for all converge to a probabilistic right-continuous function which measurably depends on all in that sens

for an arbitrary continuous bounded function . It follows that for all

where is the basis of the kernel , that is

 Denote the basis of the kernel by and let

Suppose there exists a function and a kernel on (*E*,) such that for all

For convenience, we put . Note that based on (5) and (6) for all

Let's demand that

We will assume that

From this, in particular, it follows that

Denote by and finally assume

W. Feller introduced the very important notion of direct Riemann integrability.

Namely, a family of functions on , that depend on a small parameter is called directly Riemann-integrable if the series

Under these conditions, the improper integral

is the limit of the integral sums constructed for a direct partition (hence the name) of the semi-axis uniformly on for all that is

where , in contrast to the improper Riemann integral as limit of integrals over finite intervals.

That is why such a function is called directly Riemann-integrable.

The class of directly integrable functions is sufficiently large although smaller than the class of all absolutely integrable functions. In particular, it contains all monotone bounded integrable functions. But what is of fundamental importance is that this class is closed with respect to convolution with any finite non-negative measure. Particularly, if a function is directly Riemann-integrable and is a finite non-negative measure, then the function

is also directly Riemann-integrable.

Let the distribution function be non-lattice for all and there be a limit

Each kernel naturally generates a linear operator that operates in Banach space bounded -measurable function with a norm by a formula

Denote by and the operators corresponding to the kernels and

Thus we have proved the following theorem.

**Theorem**. Let in conditions (2), (5), (6), (7), (8), (9),(10),(11),(12) for all the probability distribution be non-lattice, then

for all .

**Conclusion.**

The asymptotics of the solution of the Markov renewal equation when the basis of the kernel close to the singular kernel on a given measurable phase space (*E*,) was studied in [2].The main result of that study was formulated in the form of a theorem. At the same time, severe restrictions were imposed. Uniform convergence on was required. In this paper, we prove a similar statement under weaker assumptions, namely, it is sufficient that the conditions of the theorem are satisfied for all . For this, a completely different idea of proof is used.

**References.**

1. S. Degtyar, Markov renewal limit theorems.Theory of Probability and Mathematical Statistics, 76, pp. 33--40, 2008.
2. V. Shurenkov, and S. Degtyar, Markov renewal theorems in scheme of arrays, Asymptotic Analysis of Random Evolution, pp. 270--305, 1994.
3. V. M. Shurenkov, Ergodic Theorems and Related Problems, English transl., VSP International Science Publishers, Utrecht, 1998. MR1690361 (2000i:60002).